Finite Mathematics

for Business, Economics, Life Sciences, and Social Sciences

FOURTEENTH EDITION

Raymond A. Barnett • Michael R. Ziegler
Karl E. Byleen • Christopher J. Stocker

fourteenth edition

FINITE MATHEMATICS

for Business, Economics,
Life Sciences, and
Social Sciences

GLOBAL EDITION

RAYMOND A. BARNETT Merritt College

MICHAEL R. ZIEGLER Marquette University

KARL E. BYLEEN Marquette University

CHRISTOPHER J. STOCKER Marquette University

Pearson

CONTENTS

PREFACE

The fourteenth edition of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in finite mathematics for students who have had one to two years of high school algebra or the equivalent. The book's overall approach, refined by the authors' experience with large sections of college freshmen, addresses the challenges of teaching and learning when prerequisite knowledge varies greatly from student to student.

Note that Chapters 1–8 of this text also appear in *College Mathematics for Business, Economics, Life Sciences, and Social Sciences,* by the same author team. The College Mathematics text also contains coverage of applied calculus topics.

The authors had three main goals in writing this text:

1. To write a text that students can easily comprehend

2. To make connections between what students are learning and how they may apply that knowledge

3. To give flexibility to instructors to tailor a course to the needs of their students.

Many elements play a role in determining a book's effectiveness for students. Not only is it critical that the text be accurate and readable, but also, in order for a book to be effective, aspects such as the page design, the interactive nature of the presentation, and the ability to support and challenge all students have an incredible impact on how easily students comprehend the material. Here are some of the ways this text addresses the needs of students at all levels:

- Page layout is clean and free of potentially distracting elements.

- Matched Problems that accompany each of the completely worked examples help students gain solid knowledge of the basic topics and assess their own level of understanding before moving on.

- Review material (Appendix A, B, and Chapter 1) can be used judiciously to help remedy gaps in prerequisite knowledge.

- A Diagnostic Prerequisite Test prior to Chapter 1 helps students assess their skills, while the Basic Algebra Review in Appendix B provides students with the content they need to remediate those skills.

- Explore and Discuss problems lead the discussion into new concepts or build upon a current topic. They help students of all levels gain better insight into the mathematical concepts through thought-provoking questions that are effective in both small and large classroom settings.

- Instructors are able to easily craft homework assignments that best meet the needs of their students by taking advantage of the variety of types and difficulty levels of the exercises. Exercise sets at the end of each section consist of a Skills Warm-up (four to eight problems that review prerequisite knowledge specific to that section) followed by problems divided into categories A, B, and C by level of difficulty, with level-C exercises being the most challenging.

- The MyLab Math course for this text is designed to help students help themselves and provide instructors with actionable information about their progress. The immediate feedback students receive when doing homework and practice in MyLab Math is invaluable, and the easily accessible eText enhances student learning in a way that the printed page sometimes cannot.

- Most important, all students get substantial experience in modeling and solving real-world problems through application examples and exercises chosen from

business and economics, life sciences, and social sciences. Great care has been taken to write a book that is mathematically correct, with its emphasis on computational skills, ideas, and problem solving rather than mathematical theory.

■ Finally, the choice and independence of topics make the text readily adaptable to a variety of courses.

New to This Edition

Fundamental to a book's effectiveness is classroom use and feedback. Now in its fourteenth edition, this text has had the benefit of a substantial amount of both. Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors. Additionally, we made the following improvements in this edition:

■ Redesigned the text in full color to help students better use it and to help motivate students as they put in the hard work to learn the mathematics (because let's face it—a more modern looking book has more appeal).

■ Updated graphing calculator screens to TI-84 Plus CE (color) format.

■ Added *Reminder* features in the side margin to either remind students of a concept that is needed at that point in the book or direct the student back to the section in which it was covered earlier.

■ Updated data in examples and exercises. Many modern and student-centered applications have been added to help students see the relevance of the content.

■ Analyzed aggregated student performance data and assignment frequency data from MyLab Math for the previous edition of this text. The results of this analysis helped improve the quality and quantity of exercises that matter the most to instructors and students.

■ Rewrote and simplified the treatment of cost, revenue, and profit in Section 1.1.

■ Added 611 new exercises throughout the text.

■ Moved the seldom-used chapter "Games and Decisions" online to www.pearsonglobaleditions.com. Note that all of the resources that formerly accompanied this chapter are still available. They are housed within MyLab Math.

New to MyLab Math

Many improvements have been made to the overall functionality of MyLab Math since the previous edition. However, beyond that, we have also increased and improved the content specific to this text.

■ Instructors now have **more exercises** than ever to choose from in assigning homework. Most new questions are application-oriented. There are approximately 3,340 assignable exercises in MyLab Math for this text. New exercise types include:

• **Additional Conceptual Questions** provide support for assessing concepts and vocabulary. Many of these questions are application-oriented.

• **Setup & Solve** exercises require students to show how they set up a problem as well as the solution, better mirroring what is required of students on tests.

■ The **Guide to Video-Based Assignments** shows which MyLab Math exercises can be assigned for each video. (All videos are also assignable.) This resource is handy for online or flipped classes.

■ The *Note-Taking Guide* provides support for students as they take notes in class. The Guide includes definitions, theorems, and statements of examples but has blank space for students to write solutions to examples and sample problems. The Note-Taking Guide corresponds to the Lecture PowerPoints that accompany the text. The Guide can be downloaded in PDF or Word format from within MyLab Math.

- A full suite of **Interactive Figures** has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They have been designed to be used in lecture as well as by students independently.
- **Study Skills Modules** help students with the life skills that can make the difference between passing and failing.
- **MathTalk and StatTalk videos** highlight applications of the content of the course to business. The videos are supported by assignable exercises.
- The *Graphing Calculator Manual* and *Excel Spreadsheet Manual*, both specific to this course, have been updated to support the TI-84 Plus CE (color edition) and Excel 2016, respectively. Both manuals also contain additional topics to support the course. These manuals are within the Tools for Success tab.
- MyLab Math now contains a downloadable **Instructor's Answers document—** *with all answers in one place*. (This augments the downloadable *Instructor's Solutions Manual*, which contains even-numbered solutions.)

Trusted Features

- **Emphasis and Style**—As was stated earlier, this text is written for student comprehension. To that end, the focus has been on making the book both mathematically correct and accessible to students. Most derivations and proofs are omitted, except where their inclusion adds significant insight into a particular concept as the emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. General concepts and results are typically presented only after particular cases have been discussed.
- **Design**—One of the hallmark features of this text is the clean, straightforward design of its pages. Navigation is made simple with an obvious hierarchy of key topics and a judicious use of call-outs and pedagogical features. A functional use of color improves the clarity of many illustrations, graphs, and explanations, and guides students through critical steps (see pages 37 and 38).
- **Examples**—More than 300 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are annotated using blue text to the right of each step, and the problem-solving steps are clearly identified. To give students extra help in working through examples, dashed boxes are used to enclose steps that are usually performed mentally and rarely mentioned in other books (see Example 7 on page 29). Though some students may not need these additional steps, many will appreciate the fact that the authors do not assume too much in the way of prior knowledge.
- **Matched Problems**—Each example is followed by a similar Matched Problem for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.
- **Explore and Discuss**—Most every section contains Explore and Discuss problems at appropriate places to encourage students to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text (see pages 36 and 39). This serves to foster critical thinking and communication skills. The Explore and Discuss material can be used for in-class discussions or out-of-class group activities and is effective in both small and large class settings.
- **Exercise Sets**—The book contains over 4,200 carefully selected and graded exercises. Many problems have multiple parts, significantly increasing the total number of exercises. Writing exercises, indicated by the icon ✏, provide

students with an opportunity to express their understanding of the topic in writing. Answers to all odd-numbered problems are in the back of the book. Exercises are paired so that consecutive odd- and even-numbered exercises are of the same type and difficulty level. Exercise sets are structured to facilitate crafting just the right assignment for students:

- **Skills Warm-up** exercises, indicated by **W**, review key prerequisite knowledge.
- **Graded exercises**: Levels **A** (routine, easy mechanics), **B** (more difficult mechanics), and **C** (difficult mechanics and some theory) make it easy for instructors to create assignments that are appropriate for their classes.
- **Applications** conclude almost every exercise set. These exercises are labeled with the type of application to make it easy for instructors to select the right exercises for their audience.

- **Applications**—A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Index of Applications at the back of the book). Almost every exercise set contains application problems, including applications from business and economics, life sciences, and social sciences. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems inspired by professional journals and books. No specialized experience is required to solve any of the application problems.

- **Graphing Calculator and Spreadsheets**—Although access to a graphing calculator or spreadsheets is not assumed, it is likely that many students will want to make use of this technology. To assist these students, optional graphing calculator and spreadsheet activities are included in appropriate places. These include brief discussions in the text, examples or portions of examples solved on a graphing calculator or spreadsheet, and exercises for the students to solve. For example, regression techniques on a graphing calculator are used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing calculator material is clearly identified with the icon ⌨ and can be omitted without loss of continuity, if desired. Graphing calculator screens displayed in the text are actual output from the TI-84 Plus CE (color version) graphing calculator.

Additional Pedagogical Features

The following features, while helpful to any student, are particularly helpful to students enrolled in a large classroom setting where access to the instructor is more challenging or just less frequent. These features provide much-needed guidance for students as they tackle difficult concepts.

- **Call-out boxes** highlight important definitions, results, and step-by-step processes (see pages 40, 47, and 105).
- **Caution** statements appear throughout the text where student errors often occur (see pages 28 and 93).
- **Conceptual Insights**, appearing in nearly every section, often make explicit connections to previous knowledge but sometimes encourage students to think beyond the particular skill they are working on and attain a more enlightened view of the concepts at hand (see pages 36 and 47).

- **Diagnostic Prerequisite Test**, located on page 17, provides students with a tool to assess their prerequisite skills prior to taking the course. The **Basic Algebra Review**, in Appendix B, provides students with seven sections of content to help them remediate in specific areas of need. Answers to the Diagnostic Prerequisite Test are at the back of the book and reference specific sections in the Appendix A, B, and Chapter 1 for students to use for remediation.

- **Chapter Reviews**—Often it is during the preparation for a chapter exam that concepts gel for students, making the chapter review material particularly important. The chapter review sections in this text include a comprehensive summary of important terms, symbols, and concepts, keyed to completely worked examples, followed by a comprehensive set of Review Exercises. Answers to Review Exercises are included at the back of the book; each answer contains a reference to the section in which that type of problem is discussed so students can remediate any deficiencies in their skills on their own.

Content

The text begins with the development of a library of elementary functions in Chapter 1, including their properties and applications. Many students willbe familiar with most, if not all, of the material in this introductory chapter. Depending on students' preparation and the course syllabus, an instructor has several options for using the first chapter and appendices including the following:

 (i) Skip Chapter 1 and refer to it only as necessary later in the course;
 (ii) Cover Appendix A quickly in the first week of the course, emphasizing price–demand equations, price–supply equations, and linear regression, but skip Chapter 1;
(iii) Cover Appendix A and Chapter 1 systematically before moving on to other chapters.

The material in Finite Mathematics can be thought of as four units:

 1. Mathematics of finance (Chapter 2)
 2. Linear algebra, including matrices, linear systems, and linear programming (Chapters 3, 4, and 5)
 3. Probability and statistics (Chapters 6 and 7)
 4. Applications of linear algebra and probability to Markov chains and game theory (Chapters 8 and 10)

The first three units are independent of each other, while the fourth unit is dependent on some of the earlier chapters. (Markov chains requires Chapters 3 and 7; game theory requires Chapters 3–5 and 7).

Chapter 2 presents a thorough treatment of simple and compound interest and present and future value of ordinary annuities. Appendix C.1 addresses arithmetic and geometric sequences and can be covered in conjunction with this chapter, if desired.

Chapter 3 covers linear systems and matrices with an emphasis on using row operations and Gauss–Jordan elimination to solve systems and to find matrix inverses. This chapter also contains numerous applications of mathematical modeling using systems and matrices. To assist students in formulating solutions, all answers at the back of the book for application exercises in Sections 3.3, 3.5, and the chapter Review Exercises contain both the mathematical model and its solution. The row operations discussed in Sections 3.2 and 3.3 are required for the simplex method in Chapter 5. Matrix multiplication, matrix inverses, and systems of equations are required for Markov chains in Chapter 8.

Chapters 4 and 5 provide a broad and flexible coverage of linear programming. Chapter 5 covers two-variable graphing techniques. Instructors who wish to emphasize linear programming techniques can cover the basic simplex method in Sections 5.1 and 5.2 and then discuss either or both of the following: the dual method (Section 5.3)

and the big M method (Section 5.4). Those who want to emphasize modeling can discuss the formation of the mathematical model for any of the application examples in Sections 5.2–5.4, and either omit the solution or use software to find the solution. To facilitate this approach, all answers at the back of the book for application exercises in Sections 5.2–5.4 and the chapter Review Exercises contain both the mathematical model and its solution. The simplex and dual solution methods are required for portions of Chapter 10.

Chapter 6 provides a foundation for probability with a treatment of logic, sets, and counting techniques.

Chapter 7 covers basic probability, including Bayes' formula and random variables.

Chapter 9 deals with basic descriptive statistics and more advanced probability distributions, including the important normal distribution. Appendix C.3 contains a short discussion of the binomial theorem that can be used in conjunction with the development of the binomial distribution in Section 10.4.

Chapters 8 and 10 tie together concepts developed in earlier chapters and apply them to interesting topics. A study of Markov chains (Chapter 8) or game theory (Chapter 10) provides an excellent unifying conclusion to a finite mathematics course.

Appendix B contains a concise review of basic algebra that may be covered as part of the course or referenced as needed. As mentioned previously, Appendix C (online at www.pearsonglobaleditions.com) contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Acknowledgments

David Busekist, *Southeastern Louisiana University*
Kristen A. Ceballos, *Southern Illinois University, Carbondale*
Brandi L. Cline, *Lone Star College, Tomball*
James Colton, *Shaker High School*
Natalia Gavryshova, *College of San Mateo*
Liz Hawk, *Eastbrook High School*
Natalie Herndon, *Southwest Tennessee State Univerity*
Alan Kunkle, *Ivy Tech Community College, Kokomo*
Steve Lam, *Guam Community College*
Mari M. Menard, *Lone Star College, Kingwood*
Faramarz Mokhtari, *Montgomery College, Rockville*
Jimmy Mopecha, *Delaware Valley University*
Mike Panahi, *El Centro College*
Terri Seiver, *San Jacinto College*
Jack Wagner, *Fordham University*
Dennis Wolf, *Indiana University, South Bend*
Dinesh S. Yadav, *Richland College*

We also express our thanks to John Samons and Damon Demas for providing a careful and thorough accuracy check of the text, problems, and answers. Our thanks to Garret Etgen, John Samons, Salvatore Sciandra, Victoria Baker, Ben Rushing, and Stela Pudar-Hozo for developing the supplemental materials so important to the success of a text. And finally, thanks to all the people at Pearson and Integra who contributed their efforts to the production of this book.

Acknowledgments for the Global Edition

Pearson would like to thank the following people for their contributions to the Global Edition of this book.

Contributors
Veena Dhingra
Katarzyna Zuleta Estrugo, *École Polytechnique Fédérale de Lausanne*

Reviewers
Sunil Jacob John, *National Institute of Technology Calicut*
Mohammad Kacim, *Holy Spirit University of Kaslik*
Seifedine Kadry, *Beirut Arab University*
M. Sankar, *Presidency University*

We also express our gratitude to Vinod Ramachandran and Tibor Vörös for providing their feedback on how to enhance this book by participating in the survey.

MyLab Math Online Course

for *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences, 14e* (access code required)

MyLab™ Math is available to accompany Pearson's market-leading text offerings. To give students a consistent tone, voice, and teaching method, each text's flavor and approach are tightly integrated throughout the accompanying MyLab Math course, making learning the material as seamless as possible.

Preparedness

One of the biggest challenges in applied math courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation and key-concept review.

NEW! Study Skills Modules

Study skills modules help students with the life skills that can make the difference between passing and failing.

Developing Deeper Understanding

MyLab Math provides content and tools that help students build a deeper understanding of course content than would otherwise be possible.

Exercises with Immediate Feedback

Homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers and includes the optional learning aids Help Me Solve This, View an Example, videos, and/or the eText.

NEW! Additional Conceptual Questions

Additional Conceptual Questions provide support for assessing concepts and vocabulary. Many of these questions are application-oriented. They are clearly labeled "Conceptual" in the Assignment Manager.

NEW! Setup & Solve Exercises

These exercises require students to show how they set up a problem as well as the solution, better mirroring what is required on tests.

Solve the linear system by the Gauss-Jordan elimination method.

$$\begin{cases} x + y - z = -5 \\ -x + 4y + 16z = 7 \\ -4x + y + 4z = -7 \end{cases}$$

Without changing the order of any rows or columns, write a matrix that represents the system.

$$\begin{bmatrix} 1 & 1 & -1 & -5 \\ -1 & 4 & 16 & 7 \\ -4 & 1 & 4 & -7 \end{bmatrix}$$

(Do not simplify. Type an integer or simplified fraction for each matrix element.)

The solution of the system is $x = \dfrac{7}{3}$, $y = -\dfrac{27}{5}$, $z = \dfrac{29}{15}$.

(Simplify your answers. Type integers or fractions.)

NEW! Interactive Figures

A full suite of Interactive Figures has been added to support teaching and learning. The figures illustrate key concepts and allow manipulation. They are designed to be used in lecture as well as by students independently.

less than or equal to

5

	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9
	5	6	7	8	9	10
	6	7	8	9	10	11
	7	8	9	10	11	12

The probability that the sum of the dots is less than or equal to 5 is

$$\frac{10}{36} = 0.278.$$
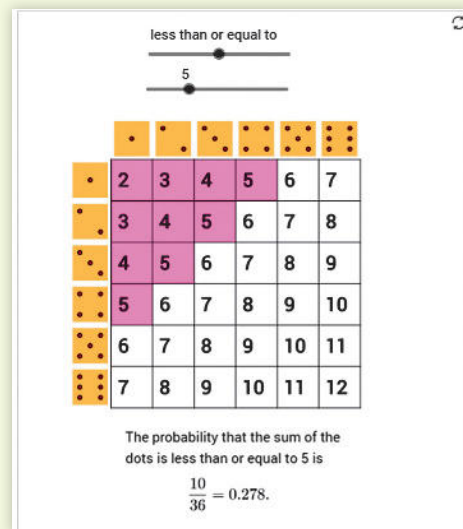
Instructional Videos

Every example in the text has an instructional video tied to it that can be used as a learning aid or for self-study. MathTalk videos were added to highlight business applications to the course content, and a Guide to Video-Based Assignments shows which MyLab Math exercises can be assigned for each video.

NEW! Note-Taking Guide (downloadable)

These printable sheets, developed by Ben Rushing (Northwestern State University) provide support for students as they take notes in class. They include preprinted definitions, theorems, and statements of examples but have blank space for students to write solutions to examples and sample problems. The *Note-Taking Guide* corresponds to the Lecture PowerPoints that accompany the text. The *Guide* can be downloaded in PDF or Word format from within MyLab Math from the Tools for Success tab.

Graphing Calculator and Excel Spreadsheet Manuals (downloadable)

Graphing Calculator Manual by Chris True, University of Nebraska
Excel Spreadsheet Manual by Stela Pudar-Hozo, Indiana University–Northwest
These manuals, both specific to this course, have been updated to support the TI-84 Plus CE (color edition) and Excel 2016, respectively. Instructions are ordered by mathematical topic. The files can be downloaded from within MyLab Math from the Tools for Success tab.

Student's Solutions Manual (downloadable)

Written by John Samons (Florida State College), the *Student's Solutions Manual* contains worked-out solutions to all the odd-numbered exercises. This manual can be downloaded from within MyLab Math within the Chapter Contents tab.

A Complete eText

Students get unlimited access to the eText within any MyLab Math course using that edition of the textbook.

Supporting Instruction

MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve students' results at every turn and unparalleled flexibility to create a course tailored to you and your students.

Learning Catalytics™

Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics. Access pre-built exercises created specifically for this course.
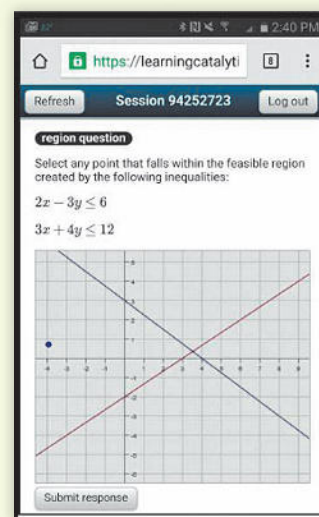
PowerPoint® Lecture Slides (downloadable)

Classroom presentation slides feature key concepts, examples, and definitions from this text. They are designed to be used in conjunction with the Note-Taking Guide that accompanies the text. They can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearsonglobaleditions.com**.

Learning Worksheets

Written by Salvatore Sciandra (Niagara County Community College), these worksheets include key chapter definitions and formulas, followed by exercises for students to practice in class, for homework, or for independent study. They are downloadable as PDFs or Word documents from within MyLab Math.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality such as item analysis and a reporting dashboard to allow you to efficiently manage your course. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.
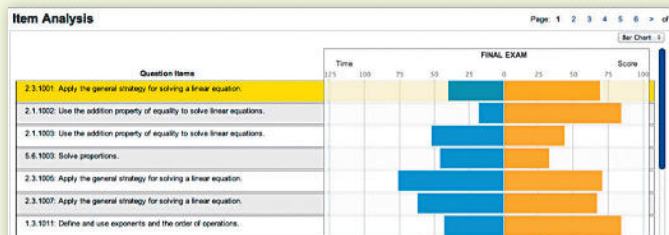
TestGen®

TestGen® (**www.pearson.com/testgen**) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog, **www.pearsonglobaleditions.com**. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual (downloadable)

Written by Garret J. Etgen (University of Houston) and John Samons (Florida State College), the *Instructor's Solutions Manual* contains worked-out solutions to all the even-numbered exercises. It can be downloaded from within MyLab Math or from Pearson's online catalog, **www.pearsonglobaleditions.com**.

Accessibility

Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, **www.pearson.com/mylab/math/accessibility**.

Diagnostic Prerequisite Test

Work all of the problems in this self-test without using a calculator. Then check your work by consulting the answers in the back of the book. Where weaknesses show up, use the reference that follows each answer to find the section in the text that provides the necessary review.

1. Replace each question mark with an appropriate expression that will illustrate the use of the indicated real number property:

   (A) Commutative $(\,\cdot\,)$: $x(y + z) = ?$

   (B) Associative $(+)$: $2 + (x + y) = ?$

   (C) Distributive: $(2 + 3)x = ?$

Problems 2–6 refer to the following polynomials:

   (A) $3x - 4$ (B) $x + 2$

   (C) $2 - 3x^2$ (D) $x^3 + 8$

2. Add all four.

3. Subtract the sum of (A) and (C) from the sum of (B) and (D).

4. Multiply (C) and (D).

5. What is the degree of each polynomial?

6. What is the leading coefficient of each polynomial?

In Problems 7 and 8, perform the indicated operations and simplify.

7. $5x^2 - 3x[4 - 3(x - 2)]$

8. $(2x + y)(3x - 4y)$

In Problems 9 and 10, factor completely.

9. $x^2 + 7x + 10$ 10. $x^3 - 2x^2 - 15x$

11. Write 0.35 as a fraction reduced to lowest terms.

12. Write $\dfrac{7}{8}$ in decimal form.

13. Write in scientific notation:

   (A) 4,065,000,000,000   (B) 0.0073

14. Write in standard decimal form:

   (A) 2.55×10^8 (B) 4.06×10^{-4}

15. Indicate true (T) or false (F):

   (A) A natural number is a rational number.

   (B) A number with a repeating decimal expansion is an irrational number.

16. Give an example of an integer that is not a natural number.

In Problems 17–24, simplify and write answers using positive exponents only. All variables represent positive real numbers.

17. $6(xy^3)^5$ 18. $\dfrac{9u^8v^6}{3u^4v^8}$

19. $(2 \times 10^5)(3 \times 10^{-3})$ 20. $(x^{-3}y^2)^{-2}$

21. $u^{5/3}u^{2/3}$ 22. $(9a^4b^{-2})^{1/2}$

23. $\dfrac{5^0}{3^2} + \dfrac{3^{-2}}{2^{-2}}$ 24. $(x^{1/2} + y^{1/2})^2$

In Problems 25–30, perform the indicated operation and write the answer as a simple fraction reduced to lowest terms. All variables represent positive real numbers.

25. $\dfrac{a}{b} + \dfrac{b}{a}$ 26. $\dfrac{a}{bc} - \dfrac{c}{ab}$

27. $\dfrac{x^2}{y} \cdot \dfrac{y^6}{x^3}$ 28. $\dfrac{x}{y^3} \div \dfrac{x^2}{y}$

29. $\dfrac{\dfrac{1}{7 + h} - \dfrac{1}{7}}{h}$ 30. $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

31. Each statement illustrates the use of one of the following real number properties or definitions. Indicate which one.

Commutative $(+, \cdot)$	Associative $(+, \cdot)$	Distributive
Identity $(+, \cdot)$	Inverse $(+, \cdot)$	Subtraction
Division	Negatives	Zero

   (A) $(-7) - (-5) = (-7) + [-(-5)]$

   (B) $5u + (3v + 2) = (3v + 2) + 5u$

   (C) $(5m - 2)(2m + 3) = (5m - 2)2m + (5m - 2)3$

   (D) $9 \cdot (4y) = (9 \cdot 4)y$

   (E) $\dfrac{u}{-(v - w)} = \dfrac{u}{w - v}$

   (F) $(x - y) + 0 = (x - y)$

32. Round to the nearest integer:

   (A) $\dfrac{17}{3}$ (B) $-\dfrac{5}{19}$

33. Multiplying a number x by 4 gives the same result as subtracting 4 from x. Express as an equation, and solve for x.

34. Find the slope of the line that contains the points $(3, -5)$ and $(-4, 10)$.

35. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the x axis.

36. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the y axis.

In Problems 37–40, solve for x.

37. $x^2 = 5x$

38. $3x^2 - 21 = 0$

39. $x^2 - x - 20 = 0$

40. $-6x^2 + 7x - 1 = 0$

This page intentionally left blank

1 Functions and Graphs

Introduction

Many marine species are dependent on light from the sun. They will be found near the surface of the ocean, because light intensity decreases dramatically with depth. We use the function concept, one of the most important ideas in mathematics, to express the precise relationship between light intensity and ocean depth (see Problems 63 and 64 in Section 1.5).

The study of mathematics beyond the elementary level requires a firm understanding of a basic list of elementary functions (see the references at the back of the book). In Chapter 1, we introduce the elementary functions and study their properties, graphs, and many applications.

1.1 Functions

- Equations in Two Variables
- Definition of a Function
- Functions Specified by Equations
- Function Notation
- Applications
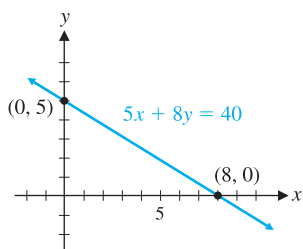
We introduce the general notion of a *function* as a correspondence between two sets. Then we restrict attention to functions for which the two sets are both sets of real numbers. The most useful are those functions that are specified by equations in two variables. We discuss the terminology and notation associated with functions, graphs of functions, and applications.

Equations in Two Variables

The graph of an equation of the form $Ax + By = C$, where A and B are not both zero, is a line. Because a line is determined by any two of its points, such an equation is easy to graph: Just plot *any* two points in its solution set and sketch the unique line through them (Fig. 1).

More complicated equations in two variables, such as $y = 9 - x^2$ or $x^2 = y^4$, are more difficult to graph. To **sketch the graph** of an equation, we plot enough points from its solution set in a rectangular coordinate system so that the total graph is apparent, and then we connect these points with a smooth curve. This process is called **point-by-point plotting**.

Figure 1

EXAMPLE 1   **Point-by-Point Plotting**   Sketch the graph of each equation.

(A) $y = 9 - x^2$ (B) $x^2 = y^4$

SOLUTION

(A) Make up a table of solutions—that is, ordered pairs of real numbers that satisfy the given equation. For easy mental calculation, choose integer values for x.

x	-4	-3	-2	-1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

After plotting these solutions, if there are any portions of the graph that are unclear, plot additional points until the shape of the graph is apparent. Then join all the plotted points with a smooth curve (Fig. 2). Arrowheads are used to indicate that the graph continues beyond the portion shown here with no significant changes in shape.
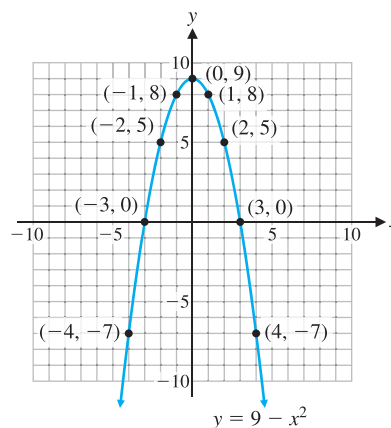
Figure 2 $y = 9 - x^2$

(B) Again we make a table of solutions—here it may be easier to choose integer values for y and calculate values for x. Note, for example, that if $y = 2$, then $x = \pm 4$; that is, the ordered pairs $(4, 2)$ and $(-4, 2)$ are both in the solution set.
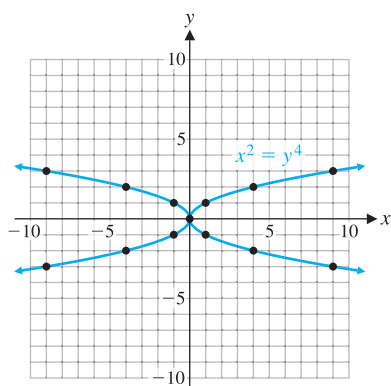
Figure 3 $x^2 = y^4$

x	± 9	± 4	± 1	0	± 1	± 4	± 9
y	-3	-2	-1	0	1	2	3

We plot these points and join them with a smooth curve (Fig. 3).

Matched Problem 1 Sketch the graph of each equation.

(A) $y = x^2 - 4$

(B) $y^2 = \dfrac{100}{x^2 + 1}$

Explore and Discuss 1

To graph the equation $y = -x^3 + 3x$, we use point-by-point plotting to obtain the graph in Figure 4.

x	y
-1	-2
0	0
1	2
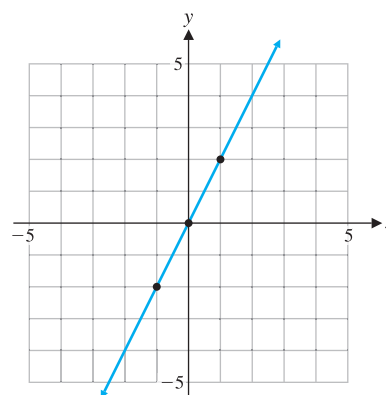
Figure 4

(A) Do you think this is the correct graph of the equation? Why or why not?

(B) Add points on the graph for $x = -2, -1.5, -0.5, 0.5, 1.5,$ and 2.

(C) Now, what do you think the graph looks like? Sketch your version of the graph, adding more points as necessary.

(D) Graph this equation on a graphing calculator and compare it with your graph from part (C).
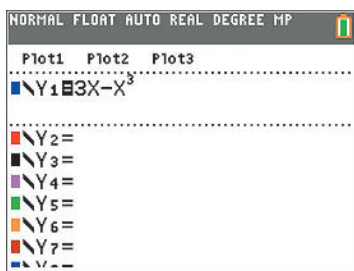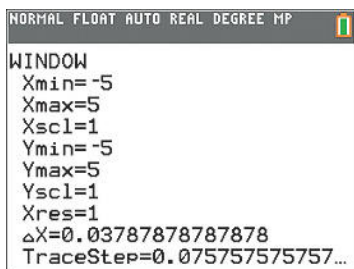
The icon in the margin is used throughout this book to identify optional graphing calculator activities that are intended to give you additional insight into the concepts under discussion. You may have to consult the manual for your graphing calculator for the details necessary to carry out these activities. For example, to graph the equation in Explore and Discuss 1 on most graphing calculators, you must enter the equation (Fig. 5A) and the window variables (Fig. 5B).

As Explore and Discuss 1 illustrates, the shape of a graph may not be apparent from your first choice of points. Using point-by-point plotting, it may be difficult to find points in the solution set of the equation, and it may be difficult to determine when you have found enough points to understand the shape of the graph. We will supplement the technique of point-by-point plotting with a detailed analysis of several basic equations, giving you the ability to sketch graphs with accuracy and confidence.

Definition of a Function

Central to the concept of function is correspondence. You are familiar with correspondences in daily life. For example,

(A)

(B)

Figure 5

To each person, there corresponds an annual income.

To each item in a supermarket, there corresponds a price.

To each student, there corresponds a grade-point average.

To each day, there corresponds a maximum temperature.

For the manufacture of x items, there corresponds a cost.

For the sale of x items, there corresponds a revenue.

To each square, there corresponds an area.

To each number, there corresponds its cube.

One of the most important aspects of any science is the establishment of correspondences among various types of phenomena. Once a correspondence is known, predictions can be made. A cost analyst would like to predict costs for various levels of output in a manufacturing process, a medical researcher would like to know the correspondence between heart disease and obesity, a psychologist would like to predict the level of performance after a subject has repeated a task a given number of times, and so on.

What do all of these examples have in common? Each describes the matching of elements from one set with the elements in a second set.

Consider Tables 1–3. Tables 1 and 2 specify functions, but Table 3 does not. Why not? The definition of the term *function* will explain.

Table 1

Domain	Range
Number	*Cube*
−2 ⟶ −8	
−1 ⟶ −1	
0 ⟶ 0	
1 ⟶ 1	
2 ⟶ 8	

Table 2

Domain	Range
Number	*Square*
−2	4
−1	1
0	0
1	
2	

Table 3

Domain	Range
Number	*Square root*
0 ⟶ 0	
	1
1	−1
	2
4	−2
	3
9	−3

DEFINITION **Function**

A **function** is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set.

The first set is called the **domain**, and the set of corresponding elements in the second set is called the **range**.

Tables 1 and 2 specify functions since to each domain value, there corresponds exactly one range value (for example, the cube of −2 is −8 and no other number). On the other hand, Table 3 does not specify a function since to at least one domain value, there corresponds more than one range value (for example, to the domain value 9, there corresponds −3 and 3, both square roots of 9).

Explore and Discuss 2

Consider the set of students enrolled in a college and the set of faculty members at that college. Suppose we define a correspondence between the two sets by saying that a student corresponds to a faculty member if the student is currently enrolled in a course taught by that faculty member. Is this correspondence a function? Discuss.
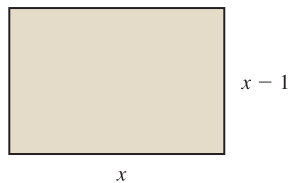
Figure 6

Functions Specified by Equations

Most of the functions in this book will have domains and ranges that are (infinite) sets of real numbers. The **graph** of such a function is the set of all points (x, y) in the Cartesian plane such that x is an element of the domain and y is the corresponding element in the range. The correspondence between domain and range elements is often specified by an equation in two variables. Consider, for example, the equation for the area of a rectangle with width 1 inch less than its length (Fig. 6). If x is the length, then the area y is given by

$$y = x(x - 1) \qquad x \geq 1$$

For each **input** x (length), we obtain an **output** y (area). For example,

$$\text{If} \quad x = 5, \quad \text{then} \quad y = \mathbf{5}(\mathbf{5} - 1) = 5 \cdot 4 = 20.$$

$$\text{If} \quad x = 1, \quad \text{then} \quad y = \mathbf{1}(\mathbf{1} - 1) = 1 \cdot 0 = 0.$$

$$\text{If} \quad x = \sqrt{5}, \quad \text{then} \quad y = \sqrt{\mathbf{5}}(\sqrt{\mathbf{5}} - 1) = 5 - \sqrt{5}$$

$$\approx 2.7639.$$

The input values are domain values, and the output values are range values. The equation assigns each domain value x a range value y. The variable x is called an *independent variable* (since values can be "independently" assigned to x from the domain), and y is called a *dependent variable* (since the value of y "depends" on the value assigned to x). In general, any variable used as a placeholder for domain values is called an **independent variable**; any variable that is used as a placeholder for range values is called a **dependent variable**.

When does an equation specify a function?

DEFINITION Functions Specified by Equations

If in an equation in two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function. The graph of such a function is just the graph of the specifying equation.

If we get more than one output for a given input, the equation does not specify a function.

EXAMPLE 2 **Functions and Equations** Determine which of the following equations specify functions with independent variable x.

(A) $4y - 3x = 8$, x a real number   (B) $y^2 - x^2 = 9$, x a real number

SOLUTION

(A) Solving for the dependent variable y, we have

$$4y - 3x = 8$$

$$4y = 8 + 3x \qquad\qquad (1)$$

$$y = 2 + \frac{3}{4}x$$

Since each input value x corresponds to exactly one output value $\left(y = 2 + \frac{3}{4}x\right)$, we see that equation (1) specifies a function.

(B) Solving for the dependent variable y, we have

$$y^2 - x^2 = 9$$
$$y^2 = 9 + x^2 \qquad (2)$$
$$y = \pm\sqrt{9 + x^2}$$

Since $9 + x^2$ is always a positive real number for any real number x, and since each positive real number has two square roots, then to each input value x there corresponds two output values ($y = -\sqrt{9 + x^2}$ and $y = \sqrt{9 + x^2}$). For example, if $x = 4$, then equation (2) is satisfied for $y = 5$ and for $y = -5$. So equation (2) does not specify a function.

Matched Problem 2   Determine which of the following equations specify functions with independent variable x.

(A) $y^2 - x^4 = 9$, x a real number   (B) $3y - 2x = 3$, x a real number

Since the graph of an equation is the graph of all the ordered pairs that satisfy the equation, it is very easy to determine whether an equation specifies a function by examining its graph. The graphs of the two equations we considered in Example 2 are shown in Figure 7.

(A) $4y - 3x = 8$ (B) $y^2 - x^2 = 9$
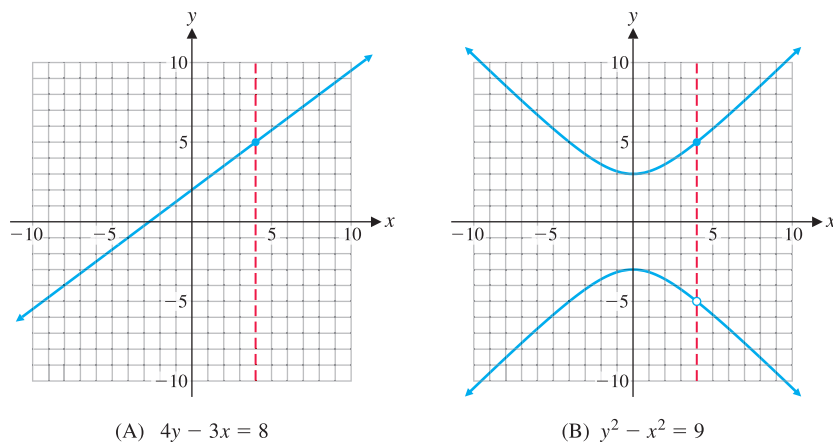
Figure 7

In Figure 7A, notice that any vertical line will intersect the graph of the equation $4y - 3x = 8$ in exactly one point. This shows that to each x value, there corresponds exactly one y value, confirming our conclusion that this equation specifies a function. On the other hand, Figure 7B shows that there exist vertical lines that intersect the graph of $y^2 - x^2 = 9$ in two points. This indicates that there exist x values to which there correspond two different y values and verifies our conclusion that this equation does not specify a function. These observations are generalized in Theorem 1.

THEOREM 1   Vertical-Line Test for a Function

An equation specifies a function if each vertical line in the coordinate system passes through, at most, one point on the graph of the equation.

If any vertical line passes through two or more points on the graph of an equation, then the equation does not specify a function.

The function graphed in Figure 7A is an example of a *linear function.* The vertical-line test implies that equations of the form $y = mx + b$, where $m \neq 0$, specify functions; they are called **linear functions**. Similarly, equations of the form $y = b$ specify functions; they are called **constant functions**, and their graphs are horizontal lines. The vertical-line test implies that equations of the form $x = a$ do not specify functions; note that the graph of $x = a$ is a vertical line.

In Example 2, the domains were explicitly stated along with the given equations. In many cases, this will not be done. Unless stated to the contrary, we shall adhere to the following convention regarding domains and ranges for functions specified by equations:

> **If a function is specified by an equation and the domain is not indicated, then we assume that the domain is the set of all real-number replacements of the independent variable (inputs) that produce real values for the dependent variable (outputs). The range is the set of all outputs corresponding to input values.**

EXAMPLE 3   **Finding a Domain**   Find the domain of the function specified by the equation $y = \sqrt{4 - x}$, assuming that x is the independent variable.

SOLUTION   For y to be real, $4 - x$ must be greater than or equal to 0; that is,

$$4 - x \geq 0$$

$$-x \geq -4$$

$$x \leq 4 \qquad \text{Sense of inequality reverses when both sides are divided by } -1.$$

Domain: $x \leq 4$ (inequality notation) or $(-\infty, 4\,]$ (interval notation)

Matched Problem 3   Find the domain of the function specified by the equation $y = \sqrt{x - 2}$, assuming x is the independent variable.

Function Notation

We have seen that a function involves two sets, a domain and a range, and a correspondence that assigns to each element in the domain exactly one element in the range. Just as we use letters as names for numbers, now we will use letters as names for functions. For example, f and g may be used to name the functions specified by the equations $y = 2x + 1$ and $y = x^2 + 2x - 3$:

$$f: \quad y = 2x + 1$$

$$g: \quad y = x^2 + 2x - 3 \qquad (3)$$

If x represents an element in the domain of a function f, then we frequently use the symbol

$$f(x)$$

in place of y to designate the number in the range of the function f to which x is paired (Fig. 8). This symbol does *not* represent the product of f and x. The symbol $f(x)$ is read as "f of x," "f at x," or "the value of f at x." Whenever we write $y = f(x)$, we assume that the variable x is an independent variable and that both y and $f(x)$ are dependent variables.

Using function notation, we can now write functions f and g in equation (3) as

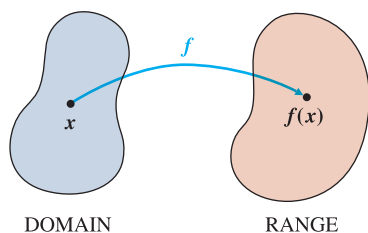$$f(x) = 2x + 1 \qquad \text{and} \qquad g(x) = x^2 + 2x - 3$$

DOMAIN   RANGE

Figure 8

Let us find $f(3)$ and $g(-5)$. To find $f(3)$, we replace x with 3 wherever x occurs in $f(x) = 2x + 1$ and evaluate the right side:

$$f(x) = 2x + 1$$

$$f(3) = 2 \cdot 3 + 1$$

$$= 6 + 1 = 7 \qquad \text{For input 3, the output is 7.}$$

Therefore,

$$f(3) = 7 \qquad \text{The function } f \text{ assigns the range value 7 to the domain value 3.}$$

To find $g(-5)$, we replace each x by -5 in $g(x) = x^2 + 2x - 3$ and evaluate the right side:

$$g(x) = x^2 + 2x - 3$$

$$g(-5) = (-5)^2 + 2(-5) - 3$$

$$= 25 - 10 - 3 = 12 \qquad \text{For input } -5, \text{ the output is 12.}$$

Therefore,

$$g(-5) = 12 \qquad \text{The function } g \text{ assigns the range value 12 to the domain value } -5.$$

It is very important to understand and remember the definition of $f(x)$:

For any element x in the domain of the function f, the symbol $f(x)$ represents the element in the range of f corresponding to x in the domain of f. If x is an input value, then $f(x)$ is the corresponding output value. If x is an element that is not in the domain of f, then f is not defined at x and $f(x)$ does not exist.

EXAMPLE 4   **Function Evaluation**   For $f(x) = 12/(x - 2)$, $g(x) = 1 - x^2$, and $h(x) = \sqrt{x - 1}$, evaluate:

(A) $f(6)$ (B) $g(-2)$ (C) $h(-2)$ (D) $f(0) + g(1) - h(10)$

SOLUTION

(A) $f(6) = \dfrac{12}{6 - 2} = \dfrac{12}{4} = 3$

(B) $g(-2) = 1 - (-2)^2 = 1 - 4 = -3$

(C) $h(-2) = \sqrt{-2 - 1} = \sqrt{-3}$

Reminder

Dashed boxes are used throughout the book to represent steps that are usually performed mentally.

But $\sqrt{-3}$ is not a real number. Since we have agreed to restrict the domain of a function to values of x that produce real values for the function, -2 is not in the domain of h, and $h(-2)$ does not exist.

(D) $f(0) + g(1) - h(10) = \dfrac{12}{0 - 2} + (1 - 1^2) - \sqrt{10 - 1}$

$$= \dfrac{12}{-2} + 0 - \sqrt{9}$$

$$= -6 - 3 = -9$$

Matched Problem 4   Use the functions in Example 4 to find

(A) $f(-2)$ (B) $g(-1)$ (C) $h(-8)$ (D) $\dfrac{f(3)}{h(5)}$

EXAMPLE 5 **Finding Domains** Find the domains of functions f, g, and h:

$$f(x) = \frac{12}{x-2} \qquad g(x) = 1 - x^2 \qquad h(x) = \sqrt{x-1}$$

SOLUTION *Domain of f:* $12/(x-2)$ represents a real number for all replacements of x by real numbers except for $x = 2$ (division by 0 is not defined). Thus, $f(2)$ does not exist, and the domain of f is the set of all real numbers except 2. We often indicate this by writing

$$f(x) = \frac{12}{x-2} \qquad x \neq 2$$

Domain of g: The domain is R, the set of all real numbers, since $1 - x^2$ represents a real number for all replacements of x by real numbers.

Domain of h: The domain is the set of all real numbers x such that $\sqrt{x-1}$ is a real number; so

$$x - 1 \geq 0$$
$$x \geq 1 \quad \text{or, in interval notation,} \quad [1, \infty)$$

Matched Problem 5 Find the domains of functions F, G, and H:

$$F(x) = x^2 - 3x + 1 \qquad G(x) = \frac{5}{x+3} \qquad H(x) = \sqrt{2-x}$$

In addition to evaluating functions at specific numbers, it is important to be able to evaluate functions at expressions that involve one or more variables. For example, the **difference quotient**

$$\frac{f(x+h) - f(x)}{h} \qquad x \text{ and } x+h \text{ in the domain of } f, h \neq 0$$

is studied extensively in calculus.

CONCEPTUAL INSIGHT

In algebra, you learned to use parentheses for grouping variables. For example,

$$2(x+h) = 2x + 2h$$

Now we are using parentheses in the function symbol $f(x)$. For example, if $f(x) = x^2$, then

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

Note that $f(x) + f(h) = x^2 + h^2 \neq f(x+h)$. That is, the function name f does not distribute across the grouped variables $(x+h)$, as the "2" does in $2(x+h)$ (see Appendix B, Section B.2).

EXAMPLE 6 **Using Function Notation** For $f(x) = x^2 - 2x + 7$, find

(A) $f(a)$

(B) $f(a+h)$

(C) $f(a+h) - f(a)$

(D) $\dfrac{f(a+h) - f(a)}{h}, \quad h \neq 0$

SOLUTION

(A) $f(a) = a^2 - 2a + 7$

(B) $f(a + h) = (a + h)^2 - 2(a + h) + 7 = a^2 + 2ah + h^2 - 2a - 2h + 7$

(C) $f(a + h) - f(a) = (a^2 + 2ah + h^2 - 2a - 2h + 7) - (a^2 - 2a + 7)$

$$= 2ah + h^2 - 2h$$

(D) $\dfrac{f(a + h) - f(a)}{h} = \dfrac{2ah + h^2 - 2h}{h} = \dfrac{h(2a + h - 2)}{h}$ Because $h \neq 0$, $\dfrac{h}{h} = 1$.

$$= 2a + h - 2$$

Matched Problem 6   Repeat Example 6 for $f(x) = x^2 - 4x + 9$.

Applications

If we reduce the price of a product, will we generate more revenue? If we increase production, will our profits rise? **Profit–loss analysis** is a method for answering such questions in order to make sound business decisions.

Here are the basic concepts of profit–loss analysis: A manufacturing company has **costs**, C, which include **fixed costs** such as plant overhead, product design, setup, and promotion and **variable costs** that depend on the number of items produced. The **revenue**, R, is the amount of money received from the sale of its product. The company takes a **loss** if $R < C$, **breaks even** if $R = C$, and has a **profit** if $R > C$. The **profit** P is equal to revenue minus cost; that is, $P = R - C$. (So the company takes a loss if $P < 0$, breaks even if $P = 0$, and has a profit if $P > 0$.) To predict its revenue, a company uses a **price–demand** function, $p(x)$, determined using historical data or sampling techniques, that specifies the relationship between the demand x and the price p. A point (x, p) is on the graph of the price–demand function if x items can be sold at a price of $\$p$ per item. (Normally, a reduction in the price p will increase the demand x, so the graph of the price–demand function is expected to go downhill as you move from left to right.) The revenue R is equal to the number of items sold multiplied by the price per item; that is, $R = xp$.

Cost, revenue, and profit can be written as functions $C(x)$, $R(x)$, and $P(x)$ of the independent variable x, the number of items manufactured and sold. The functions $C(x)$, $R(x)$, $P(x)$, and $p(x)$ often have the following forms, where a, b, m, and n are positive constants determined from the context of a particular problem:

Cost function

$$C(x) = a + bx \qquad \text{$C =$ fixed costs $+$ variable costs}$$

Price–demand function

$$p(x) = m - nx \qquad \text{x is the number of items that can be sold at $\$p$ per item}$$

Revenue function

$$R(x) = xp \qquad \text{$R =$ number of items sold $\times$ price per item}$$

$$= x(m - nx)$$

Profit function

$$P(x) = R(x) - C(x)$$

$$= x(m - nx) - (a + bx)$$

⚠ **CAUTION**   Do not confuse the price–demand function $p(x)$ with the profit function $P(x)$. Price is always denoted by the lowercase p. Profit is always denoted by the uppercase P. Note that the revenue and profit functions, $R(x)$ and $P(x)$, depend on the price–demand function $p(x)$, but $C(x)$ does not. ▲

Example 7 and Matched Problem 7 provide an introduction to profit–loss analysis.

EXAMPLE 7 **Price–Demand and Revenue**   A manufacturer of a popular digital camera whole-sales the camera to retail outlets throughout the United Kingdom. Using statistical methods, the financial department in the company produced the price–demand data in Table 4, where p is the wholesale price per camera at which x million cameras are sold. Notice that as the price goes down, the number sold goes up.

Table 4 Price–Demand

x (millions)	p($)
2	87
5	68
8	53
12	37

Using special analytical techniques (regression analysis), an analyst obtained the following price–demand function to model the Table 4 data:

$$p(x) = 94.8 - 5x \qquad 1 \le x \le 15 \qquad (4)$$

(A) Plot the data in Table 4. Then sketch a graph of the price–demand function in the same coordinate system.

(B) What is the company's revenue function for this camera, and what is its domain?

(C) Complete Table 5, computing revenues to the nearest million dollars.

(D) Plot the data in Table 5. Then sketch a graph of the revenue function using these points.

(E) Graph the revenue function on a graphing calculator.

Table 5 Revenue

x (millions)	$R(x)$ (million $)
1	90
3	
6	
9	
12	
15	

SOLUTION

(A) The four data points are plotted in Figure 9. Note that $p(1) = 89.8$ and $p(15) = 19.8$. So the graph of the price–demand function is the line through $(1, 89.8)$ and $(15, 19.8)$ (see Fig. 9).
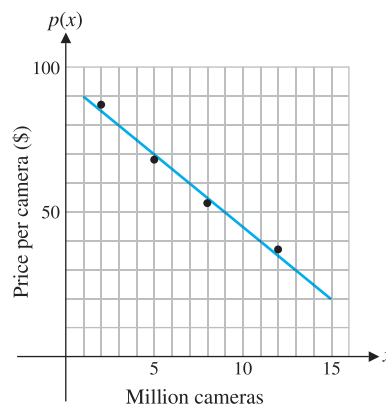

Figure 9 Price–demand

In Figure 9, notice that the model approximates the actual data in Table 4, and it is assumed that it gives realistic and useful results for all other values of x between 1 million and 15 million.

(B) $R(x) = xp(x) = x(94.8 - 5x)$ million dollars

Domain: $1 \le x \le 15$
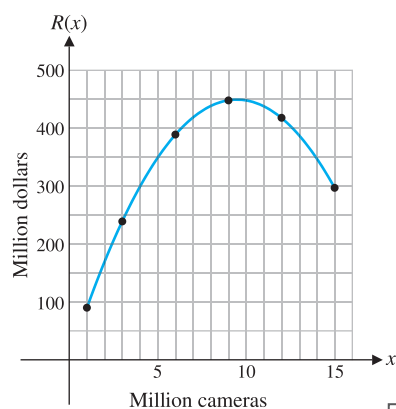
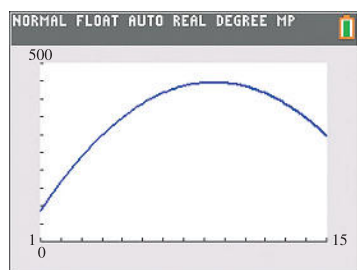[Same domain as the price–demand function, equation (4).]

Figure 10

Figure 11

(C)

Table 5 Revenue

x (millions)	R(x) (million $)
1	90
3	239
6	389
9	448
12	418
15	297

(D) The six points from Table 5 are plotted in Figure 10. The graph of the revenue function is the smooth curve drawn through those six points.

(E) Figure 11 shows the graph of $R(x) = x(94.8 - 5x)$ on a graphing calculator.

Matched Problem 7   The financial department in Example 7, using statistical techniques, produced the data in Table 6, where $C(x)$ is the cost in millions of dollars for manufacturing and selling x million cameras.

Table 6 Cost Data

x (millions)	C(x) (million $)
1	175
5	260
8	305
12	395

Using special analytical techniques (regression analysis), an analyst produced the following cost function to model the Table 6 data:

$$C(x) = 156 + 19.7x \qquad 1 \le x \le 15 \qquad (5)$$

(A) Plot the data in Table 6. Then sketch a graph of equation (5) in the same coordinate system.

(B) Using the revenue function from Example 7(B), what is the company's profit function for this camera, and what is its domain?

(C) Complete Table 7, computing profits to the nearest million dollars.

Table 7 Profit

x (millions)	P(x) (million $)
1	−86
3	
6	
9	
12	
15	

(D) Plot the data in Table 7. Then sketch a graph of the profit function using these points.

(E) Graph the profit function on a graphing calculator.